

Research paper

## Cavitation characteristics of offset-into-flow and effect of aeration

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### ABSTRACT

An experimental investigation of the cavitation characteristics of high-velocity flow over an offset-into-flow with various heights and chamfers in the presence and absence of aeration was conducted in a cavitation tunnel of the Hydraulics Laboratory, Zhejiang University of Technology in China. The 2- and 5-mm abrupt and sloping offsets into flows were employed to be models of surface irregularities. Cavitation phenomena of high-velocity flow over the various offsets were carefully observed. A range of air concentrations was considered. Variations of wall pressures downstream and upstream of the offset were measured in terms of the air concentration. The effects of air concentration and chamfer on the cavitation characteristics of the offsets were analysed. In addition, the cavitation cloud over the offset-into-flow and the turbulent boundary layer over a flat plate are compared.

**Keywords:** Air concentration, cavitation cloud, cavitation number, offset, wall pressure

### 1 Introduction

Chute or tunnel spillways for high dams involve high-velocity flow, so cavitation damage may occur if projections or depressions such as cylindrical and triangular protrusions, offsets into and away from the flow, and other irregularities during construction of flood release structures exist on the chute surface. A serious problem is cavitation damage. Hydraulic practice demonstrated that severe cavitation damage occurred even for irregularities of a few millimetres. An early case of cavitation damage was reported for Hoover Dam (Warnock 1947) within and just below its 50° vertical bend upon its initial and only operation in 1941. The velocity was as high as 45 m/s. The cavitation damage consisted of erosion of the concrete tunnel lining and underlying rock to a depth of 13.7 m and a length of 35 m, which was triggered by a bulge in the invert concrete. Another severe cavitation damage occurred in a right tunnel spillway of the Liujiaxia Hydropower Station in China (Liu 1983), arising from three 8 mm steel bars protruding on the concrete surface near the end of vertical bend of the tunnel spillway in 1972. The cavitation hole was 100 m long and 4.8 m deep.

Falvey's (1990) measurements indicate that velocities of 12–20 m/s may cause cavitation damage at local irregularities along spillway or chute surfaces. Shalnev (1951) investigated issues of cavitation damage due to surface irregularities on the blades of hydraulic turbine. His experimental data indicated that cavitation was related to the relative height of an irregularity to the velocity distribution within the boundary layer and its thickness. Ball (1959, 1963 and 1976) experimentally investigated the hydraulic consequences of surface irregularities and developed a relationship among the critical pressure, the critical velocity and its effect on cavitation inception for several types of irregularities. He suggested that a cavitation damage due to irregularities may be prevented, if the cavitation number at the irregularity was higher than the incipient cavitation number at the overflow. Holl (1960) made experiments on cavitation inception of arc and triangular irregularities, to analyse the boundary layer effect on cavitation inception. Chen (1981) suggested a numerical approach to control spillway surface irregularities based on his experimental data. Benson (1981) investigated cavitation inception of a three-dimensional

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lated irregularity. Arndt and Ippen (1968) explored the characteristics of cavitation inception of an isolated irregularity both smooth and rough surfaces. Wang and Zhou (1979) suggested a numerical method to control overflow surface singularities based on the cavitation intensity using prototype observations and model test data. Xu and Zhou (1982) analysed pressure distribution and the incipient cavitation number of abrupt and semi-arc irregularities in open-channel and pipe flows based on the theory of complex function. Liu (1983) systematically investigated the incipient cavitation number of several types of irregularities for open-channel and pipe flows, and identified the Reynolds number and the velocity profile within the boundary layer as the main effects on incipient cavitation number. He proposed that the larger the aspect ratio of irregularity was, the lower is the incipient cavitation number.

As the cavitation phenomenon is inevitable, an economic and effective measure is the aerator to force aeration within the lower-pressure cavitation zone or other locations liable to cavitation damage to prevent damage. Peterka (1953) investigated the mechanism of cavitation control by aeration. The ratios of air to water discharges in his tests ranged from 0.4% to 7.4%. He observed bursting and hammer-beating sounds due to cavitation, which reduced with increasing air discharge; the cavitation noise vanished completely as 7% air was supplied. Rasmussen's (1956) experimental tests indicated that an air–water ratio of 7% was sufficient to control cavitation for aluminum-alloy specimens. Colgate (1959) determined a correlation between cavitation inception of multiple irregularities and wall turbulence. Colgate and Legas (1972) applied aerators to control cavitation for rehabilitating cavitation damage due to irregularities in tunnel spillway at the Yellowtail dam in the USA, resulting in cavitation damage after remediation by epoxy sand grout. Field observations to control cavitation damage of a triangular irregularity by air supply to the left and right tunnel spillways at the left ski-jump spillway of flood release structures with different overflow type, head and air concentration were conducted by Deng and Ha (1988) at Wujiangdu Hydropower Station, China. The cavitation damage of a 20 mm triangular irregularity was removed as the air concentration reached 1.1% and 50 mm at 4.1%. Similar studies were carried out by Johnson (1963), Johnson (1983), Chai and Wu (1995), Wilhelms and Gulliver (2005), Li *et al.* (2007), Hocevar *et al.* (2007) or Dong *et al.* (2008a, 2008b). Regarding the cavitation characteristics of the offset-into-flow, Cai (1985) and Yuan and Zheng (1990) investigated the effect of pressure fluctuation on cavitation inception. This research aims at experimentally testing the cavitation characteristics of the offset-into-flow in presence and absence of aeration.

### Experimental facility and methodology

The test work was carried out at the Hydraulic Research Institute, Jiangsu University of Technology, China. The working sections of the tunnel system consisted mainly of a rectangular contraction, test section and expansion sections made of stainless steel plates and

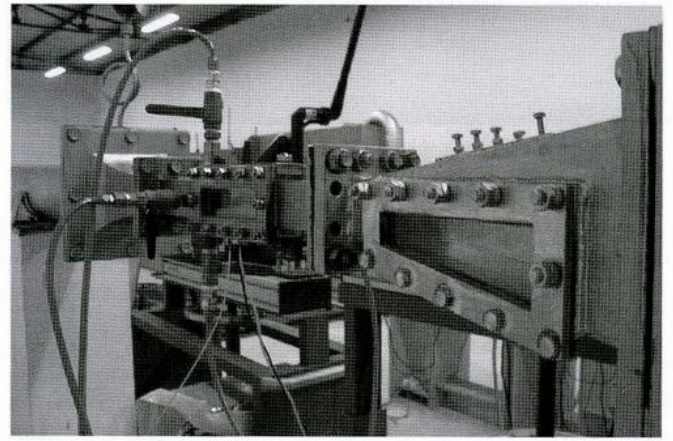


Figure 1 Experimental set-up

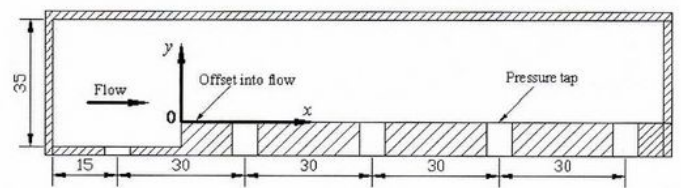


Figure 2 Locations of pressure transducers in test section of cavitation tunnel (unit: mm)

processed by computer-controlled machine (Fig. 1). The dimensions of working sections involved a cross-sectional area of  $2 \times 3.5 \text{ cm}^2$ , 50 cm long test section, the 20 cm long inflow and outflow cross-sections of the contraction section of  $15 \times 15 \text{ cm}^2$  and  $3.5 \times 4 \text{ cm}^2$  and the 40 cm long inflow and outflow of the expansion section of  $3.5 \times 4 \text{ cm}^2$  and  $15 \times 15 \text{ cm}^2$ , and observation windows. Also, there was a transition between the contraction (expansion) and the test sections. The surface irregularity was abrupt involving sloping offsets-into-flow made of stainless steel processed by computer-controlled machinery. The offset-into-flow may be divided into four types: 2 mm abrupt, 2 mm sloping, 5 mm abrupt and 5 mm sloping. Locations of pressure transducers in the test section were  $-15$ ,  $15$ ,  $45$ ,  $75$  and  $105 \text{ mm}$  from the leading offset edge (Fig. 2). The slope of the sloping offsets-into-flow was  $\tan \theta = 1/2$ .

The water discharge was measured with an UFLO2000F Ultrasonic Doppler flow meter, and the air discharge with LZB-15 and LZB-40 rotator flowmeters. MPX400D pressure transducers and the real-time data acquisition system SINO-CERA-YE6263 were used to record wall pressures. The flow velocity ranged from  $36.9$  to  $40.3 \text{ m/s}$ .

### 3 Cavitation phenomena of offset-into-flow

Cavitation phenomena of abrupt and sloping offsets-into-flow are shown in Fig. 3. Figure 3(a) and (b) shows cavitation clouds due to the 2 mm abrupt and sloping offsets, and Fig. 3 (c) and (d) due to the 5 mm abrupt and sloping elements. Accordingly, considerable cavitation phenomena of high-velocity flow over offset-into-flow are seen to occur no matter whether the offsets are abrupt or sloping. The thickness of the



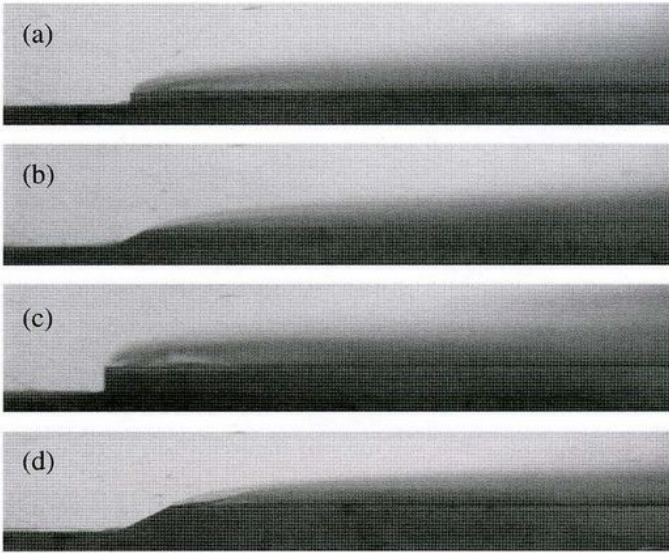


Figure 3 Cavitation clouds over offsets-into-flow (a) 2 mm abrupt, (b) 2 mm sloping, (c) 5 mm abrupt and (d) 5 mm sloping

cavitation cloud over the sloping offset was thinner than that of the abrupt of same height, especially close to the leading offset edge. As the height of the offset increases, the thickness of the cavitation cloud thickened (Fig. 3c and d).

Considering a cavitation cloud over an offset-into-flow as a turbulent boundary layer growth over a flat plate, the boundary layer momentum deficit thickness  $\delta_2$  can be expressed as

$$\delta_2 = \int_0^{\delta} \frac{u}{U_0} \left(1 - \frac{u}{U_0}\right) dy \quad (1)$$

where  $u$  is the velocity within boundary layer,  $U_0$  the approach flow velocity,  $y$  the vertical coordinate and  $\delta$  the boundary layer thickness. Eq. (1) can also be written as

$$\frac{\delta_2}{\delta} = \int_0^1 \frac{u}{U_0} \left(1 - \frac{u}{U_0}\right) d\left(\frac{y}{\delta}\right) \quad (2)$$

The velocity distribution within a turbulent boundary layer using the  $1/7$  power law is

$$\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7} \quad (3)$$

Substituting Eq. (3) into Eq. (2) gives

$$\frac{\delta_2}{\delta} = \int_0^1 \left(\frac{y}{\delta}\right)^{1/7} \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] d\left(\frac{y}{\delta}\right) = \frac{7}{72} \quad (4)$$

Based on Blasius's (1913) formula, the wall shear stress is expressed as

$$\tau_0 = 0.0225 \rho U_0^2 \left(\frac{\nu}{U_0 \delta}\right)^{1/4} \quad (5)$$

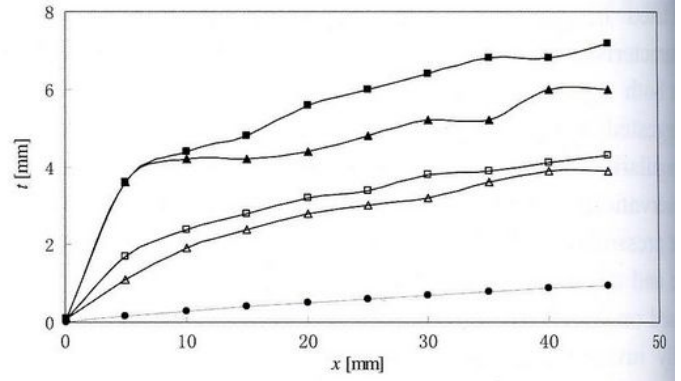


Figure 4 Comparison between cavitation cloud  $t(x)$  and boundary layer thicknesses for (▲) 2 (mm) abrupt, (△) 2 (mm) sloping, (■) 5 (mm) abrupt, (□) 5 (mm) sloping, (●) boundary layer thickness

where  $\tau_0$  = wall shear stress. The momentum integral equation of the flat plate boundary layer under zero pressure gradient

$$\frac{\tau_0}{\rho U_0^2} = \frac{d\delta_2}{dx}$$

Substituting Eqs (4) and (5) into Eq. (6) gives after integration

$$\frac{\delta}{x} = \frac{0.37}{R_x^{1/5}} \cong 9.5 \times 10^{-9} \left(\frac{1}{x}\right)$$

where  $R_x = U_0 x / \nu$  = Reynolds number of boundary layer. The streamwise variation of the turbulent boundary layer thickness  $\delta(x)$  on the smooth flat plate is obtained from Eq. (7). A comparison between the calculated boundary layer and the measured cavitation cloud thickness  $t(x)$  is shown in Fig. 4. It follows that the thickness of the cavitation cloud is considerably larger than that of the boundary layer, namely 6 to 22 and 4 to 10, respectively times thicker than the boundary layer.

#### 4 Measured pressure with and without aeration

The air concentration  $C$  is defined as

$$C = Q_a / (Q_a + Q_w)$$

where  $Q_a$  = air discharge (0.6 to 2.8 L/s) and  $Q_w$  = water discharge (22.8 to 28.3 L/s), respectively. The measured data of wall pressure in high-velocity flow over both the 2 mm abrupt and sloping offsets and the 5 mm abrupt and sloping offsets for  $C = 0.0, 2.3, 4.0, 6.0, 8.0$  and  $10.0\%$  are listed in Table 1. Taking the 2 mm abrupt offset-into-flow for example, it follows that the wall pressure without aeration is positive upstream of the offset, i.e.  $x = -15$  mm, i.e. upstream from the offset. However, the pressure downstream of the offset becomes negative, i.e.  $p = -95.3, -92.1$  kPa at  $x = 15, 45$  and  $105$  mm. The maximum wall pressure drop is located close to the leading offset edge. The wall pressure gradually decreased with  $x$ . Further, the



Table 1 Measured pressures with and without aeration

x (mm)	Wall pressure $p$ (kPa)				Air concentration $C$ (%)
	2 mm high		5 mm high		
	Abrupt offset	Sloping offset	Abrupt offset	Sloping offset	
-15	250.2	248.2	255.4	260.2	0.0
15	-97.5	-98.5	-97.5	-97.1	
45	-95.3	-96.5	-97.5	-95.4	
75	-92.1	-94.0	48.1	-80.5	
105	-78.8	-90.3	50.4	-71.2	
-15	279.8	300.5	282.1	285.1	2.3
15	-80.5	-70.6	-80.6	-70.4	
45	-73.4	-65.3	-75.3	-65.2	
75	18.7	28.5	-40.4	30.6	
105	23.4	-18.1	-55.2	-22.3	
-15	319.5	315.4	324.0	316.8	4.0
15	-60.7	-50.6	-93.5	-45.2	
45	-45.5	-45.5	-65.3	-40.5	
75	40.6	50.8	30.4	60.7	
105	28.5	8.4	-65.1	10.4	
-15	351.4	330.7	356.9	361.5	6.0
15	-40.2	-30.3	-50.2	-25.2	
45	-20.5	-15.4	-37.7	-10.4	
75	60.4	85.2	10.3	95.2	
105	50.1	35.6	-37.4	32.5	
-15	370.1	374.8	373.4	380.4	8.0
15	-20.2	-5.4	-31.5	0.0	
45	20.7	30.6	-17.2	30.5	
75	100.1	110.3	40.5	102.2	
105	78.7	50.8	-28.4	48.6	
15	389.7	380.1	394.2	391.3	10.0
15	10.3	20.5	-22.4	17.5	
45	70.2	75.4	5.3	70.2	
75	150.6	115.3	65.4	120.5	
105	90.8	63.3	-17.6	58.4	

pressures downstream of the offset at  $x = 15, 45$  and  $105$  mm increase with air concentration.

### Cavitation number of offset-into-flow

#### Cavitation number without aeration

The cavitation number is essentially a pressure coefficient, reflecting the effect of pressure change on the fluid property, expressed as

$$\sigma = \frac{h - h_v}{U_0^2 / 2g} \quad (9)$$

where  $\sigma$  is the cavitation number,  $h$  the absolute pressure head,  $h_v$  the saturated vapour pressure head related to the water temperature,  $h_v = 0.198$  m water column for  $17^\circ\text{C}$  used in the tests.

Table 2 Cavitation numbers without aeration

$x$ (mm)	Cavitation number $\sigma$			
	2 mm high		5 mm high	
	Abrupt offset	Sloping offset	Abrupt offset	Sloping offset
-15	0.4296	0.4168	0.5547	0.4968
15	0.0007	0.0001	0.0009	0.0015
45	0.0038	0.0019	0.0049	0.0043
75	0.0075	0.0049	0.0206	0.0251
105	0.0248	0.0097	0.0378	0.0376

The cavitation numbers downstream and upstream of the offset without aeration are shown in Table 2. It follows that  $\sigma$  downstream of offset-into-flow is low almost approaching  $\sigma = 0$ , whereas  $\sigma \cong 0.4$  upstream of the offset.

#### 5.2 Cavitation number of 2 mm abrupt offset-into-flow with aeration

The cavitation number of the 2 mm abrupt offset is shown in Fig. 5. It follows that  $\sigma$  downstream of the offset without aeration nearly tends to zero. However, the cavitation number downstream of the offset with aeration gradually increases with the air concentration  $C = 2.3, 4.0, 6.0, 8.0$  and  $10.0\%$ . Note that the cavitation number downstream of the test section decreases, because the measuring point was influenced by the expansion section. In the presence of aeration, the cavitation number is therefore seen to first abruptly decrease and then gradually increase.

#### 5.3 Comparison of cavitation numbers between 2 mm sloping and abrupt offsets

A comparison of the cavitation numbers between sloping and abrupt offsets 2 mm high is shown in Fig. 6. The difference of cavitation numbers between the 2 mm sloping and abrupt

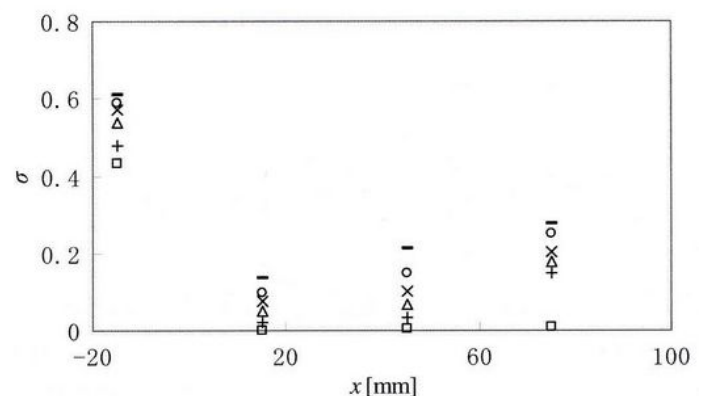


Figure 5 Cavitation number of 2 mm abrupt offset for  $C$  (%) = ( $\square$ ) 2.3, ( $\Delta$ ) 4.0, ( $\times$ ) 6.0, ( $\circ$ ) 8.0, ( $-$ ) 10.0

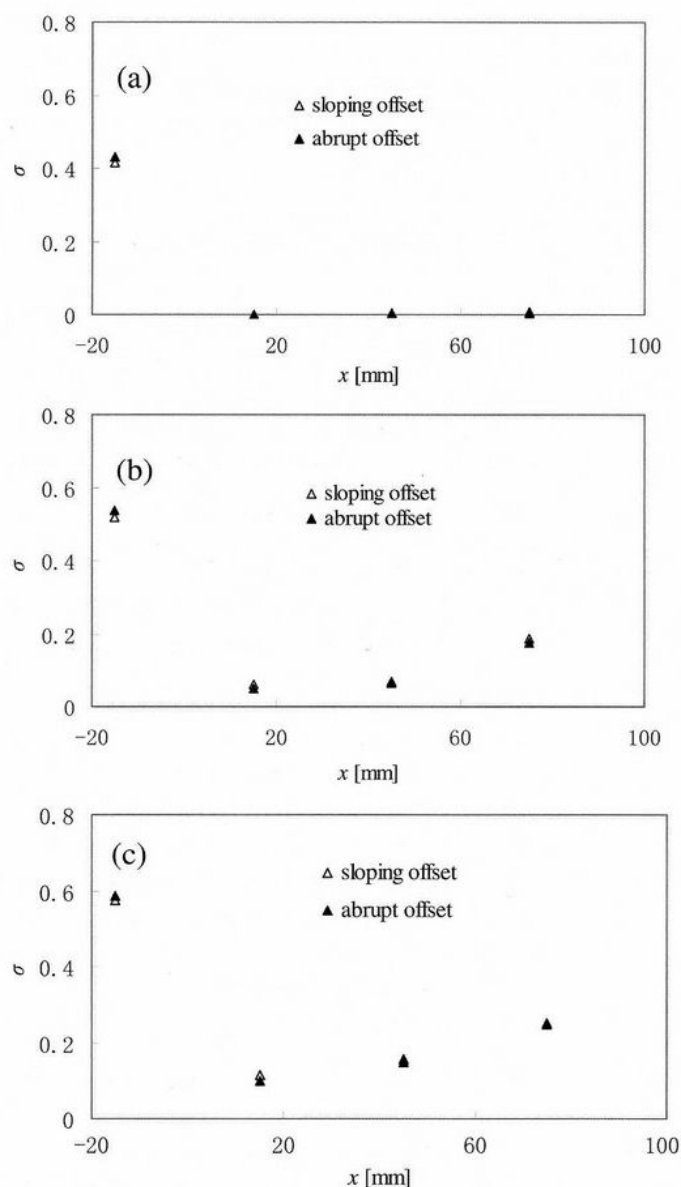


Figure 6 Comparison of cavitation number between sloping and abrupt 2 mm offsets for  $C$  (%) = (a) 0, (b) 4.0 and (c) 8.0

offsets is seen to be negligible for  $C = 2.3, 4.0, 6.0, 8.0, 10.0\%$  and for  $C = 0.0\%$ , due to the lower offset height.

#### 5.4 Cavitation number of 5 mm abrupt offset with aeration

The variation of the cavitation number for the 5 mm abrupt offset is shown in Fig. 7, which is seen to be similar to that of the 2 mm abrupt offset (Fig. 5). Accordingly, the effect of offset height on  $\sigma$  is small.

#### 5.5 Comparison of cavitation numbers between 5 mm sloping and abrupt offsets

For 5 mm abrupt offsets, the variation in the cavitation number between the sloping and the abrupt as shown in Fig. 8 differs from the 2 mm offset. The cavitation number of sloping offsets is obviously larger than that of abrupt offsets for  $C > 0$ .

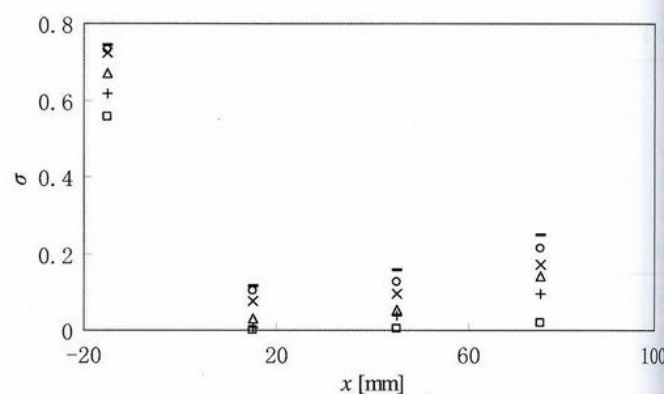


Figure 7 Cavitation number of 5 mm abrupt offset for  $C$  (%) = (a) 2.3, (b) 4.0, (c) 6.0, (d) 8.0, (e) 10.0

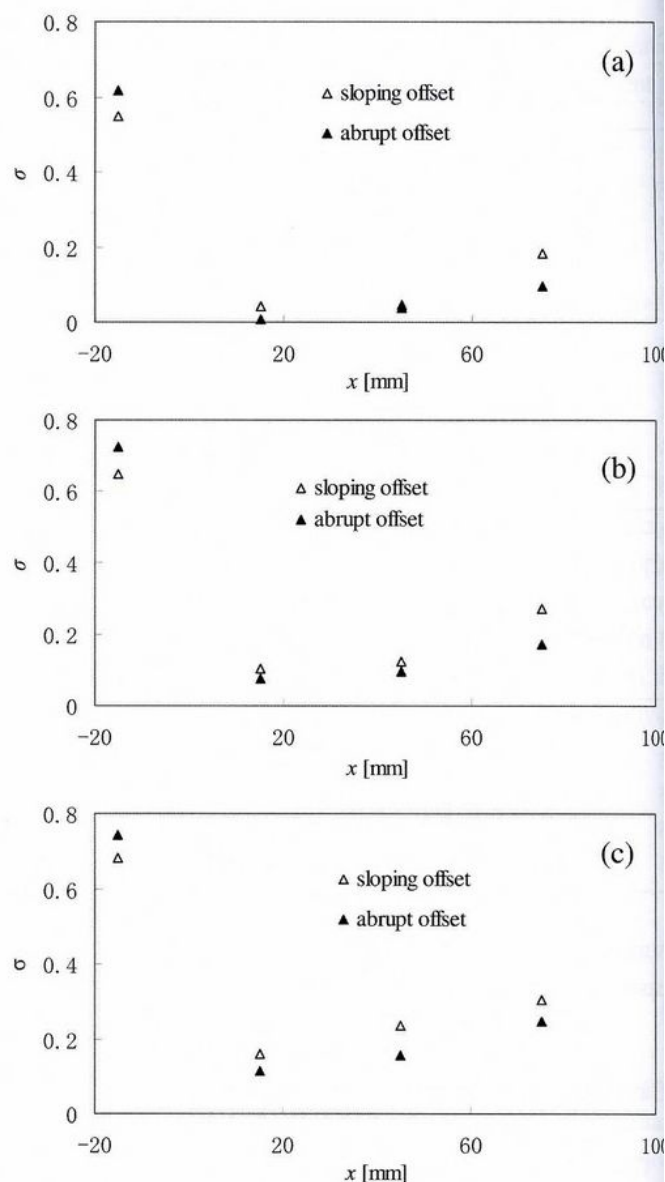


Figure 8 Comparison of cavitation number between 5 mm sloping and abrupt offsets for  $C$  [%] = (a) 2.3, (b) 6.0 and (c) 10.0

## 6 Conclusions

Based on an experimental study of the cavitation characteristics of the abrupt and sloping offsets-into-flow of various heights following conclusions are made:



- A thick cavitation cloud over the offset was generated even for an offset height of only 2 mm provided high-velocity flow in the order of 40 m/s is considered.
- Using a sloping instead of an abrupt offset did not improve the cavitation number.
- Negative pressures downstream of abrupt and sloping offsets approached  $-1.0$  atmospheric pressure in the absence of aeration.
- Aeration increases the pressure downstream of the offset considerably.
- The cavitation numbers downstream of both the abrupt and sloping offsets nearly tend to zero in the absence of aeration. However, the cavitation number downstream of the offset gradually increases with the presence of aeration.
- For 5 mm offsets, the cavitation number of the sloping offset was apparently larger than that of the abrupt in presence of aeration. In addition, the thickness of cavitation cloud over the offset is considerably larger than that of the corresponding boundary layer over a flat plate.

These test results are important relative to surface irregularity control during construction of high-head flood release structures.

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### Notation

$C$	air concentration
$p, h_v$	absolute and saturated vapour pressure head
$p_w$	wall pressure
$Q_a, Q_w$	air and water discharge
$Re_x$	Reynolds number of boundary layer
$u$	velocity within boundary layer
$U_0$	approach flow velocity
$x$	streamwise coordinate
$\delta$	thickness of boundary layer
$\delta^+$	momentum deficit thickness of boundary layer
$\sigma$	cavitation number
$\tau_w$	wall shear stress

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